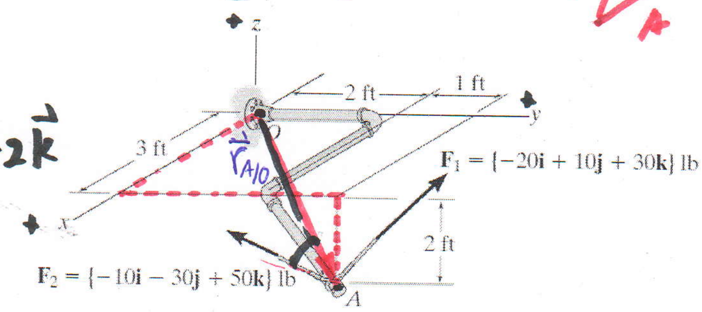
ครั้งที่ 5 22 ส.ค. 61 5/1

$$\Sigma \vec{M}_O = \vec{r}_{A/O} \times \vec{F}_1 + \vec{r}_{A/O} \times \vec{F}_2$$

$$[\vec{M}_O]_{F_2} = \vec{r}_{A/O} \times \vec{F}_2$$

4–38. Determine the moment produced by $\mathbf{F}_2$ about point O. Express the result as a Cartesian vector.



$$\vec{r}_{A/O} = \vec{r}_{OA} = 3\vec{i} + 3\vec{j} - 2\vec{k}$$

$$[\vec{M}_O]_{F_1} = \vec{r}_{A/O} \times \vec{F}_2$$

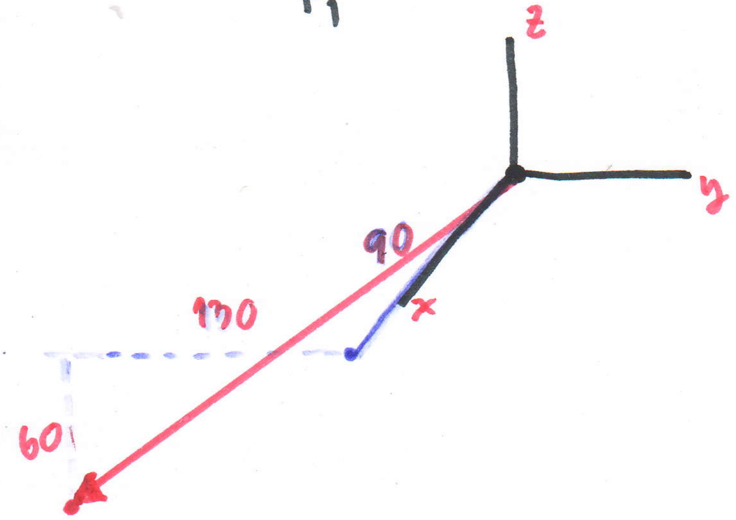
$$= [3\vec{i} + 3\vec{j} - 2\vec{k}] \times [-10\vec{i} - 30\vec{j} + 50\vec{k}]$$

$$= \left| \begin{array}{ccc|cc} i & j & k & i & j \\ 3 & 3 & -2 & 3 & 3 \\ -10 & -30 & 50 & -10 & -30 \end{array} \right|$$

$$[\vec{M}_O]_{F_1} = 150\vec{i} + 20\vec{j} - 90\vec{k} - [-30\vec{k} + 60\vec{i} + 150\vec{j}]$$

$$[\vec{M}_O]_{F_1} = 90\vec{i} - 130\vec{j} - 60\vec{k}$$

$$|\vec{M}_O|_{F_1} = \sqrt{90^2 + (-130)^2 + (-60)^2} = 169.12 \text{ lb.ft}$$



$$[\vec{M}_o]_{F_1} = \vec{r}_{A/O} \times \vec{F}_1$$

$$= [3\vec{i} + 3\vec{j} - 2\vec{k}] \times [-20\vec{i} + 10\vec{j} + 30\vec{k}]$$

$$= \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 3 & 3 & -2 & 3 & 3 \\ -20 & 10 & 30 & -20 & 10 \end{array}\right.$$

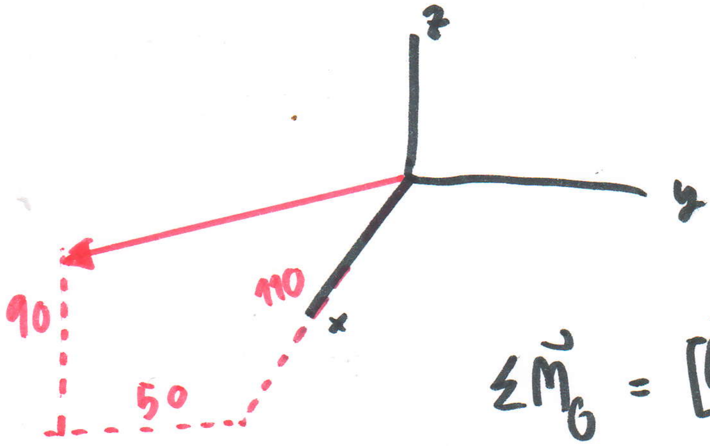
$$[\vec{M}_o]_{F_1} = 90\vec{i} + 40\vec{j} + 30\vec{k}$$

$$- [-60\vec{k} - 20\vec{i} + 90\vec{j}]$$

$$= 110\vec{i} - 50\vec{j} + 90\vec{k}$$

$$|\vec{M}_o|_{F_1} = \sqrt{110^2 + 50^2 + 90^2}$$

$$= 150.67 \quad lb.ft$$



$$\Sigma\vec{M}_o = [\vec{M}_o]_{F_1} + [\vec{M}_o]_{F_2}$$

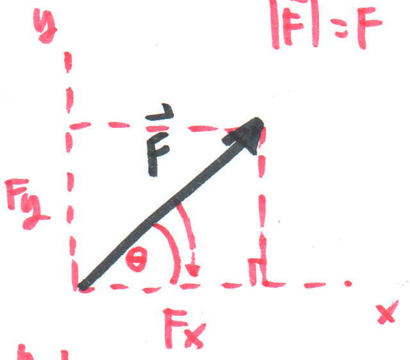
$$\Sigma\vec{M}_o = [90\vec{i} - 130\vec{j} - 60\vec{k}] + [110\vec{i} - 50\vec{j} + 90\vec{k}]$$

$$= 200\vec{i} - 180\vec{j} + 30\vec{k}$$

• Direction cosine

2D

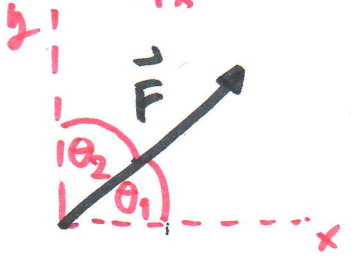
$|\vec{F}| = F$



$F_x = F\cos\theta$

$F_y = F\sin\theta$

$= F\cos(90-\theta)$



$F_x = F\cos\theta_1$

$F_y = F\cos\theta_2$

$$\vec{F} = \vec{F}_x + \vec{F}_y$$

$$= F\cos\theta_1 \vec{i} + F\cos\theta_2 \vec{j}$$

$$\vec{F} = F[\cos\theta_1 \vec{i} + \cos\theta_2 \vec{j}]$$

$$\frac{\vec{r}}{|\vec{r}|} = \vec{u}_r$$

$$\frac{\vec{F}}{F} = \cos\theta_1 \vec{i} + \cos\theta_2 \vec{j}$$

เทอมขวามือ คือ Unit vector ที่มีทิศทาง เดียวกับแรง F เราเรียกว่า Direction cosine

$$|\cos\theta_1 \vec{i} + \cos\theta_2 \vec{j}| = 1$$

$$(\cos\theta_1)^2 + (\cos\theta_2)^2 = 1$$

$$(\cos\theta_1)^2 + [\cos(90-\theta_1)]^2 = 1$$

$$(\cos\theta_1)^2 + (\sin\theta_1)^2 = 1$$

โดยปกติ เราจะกำหนด ให้

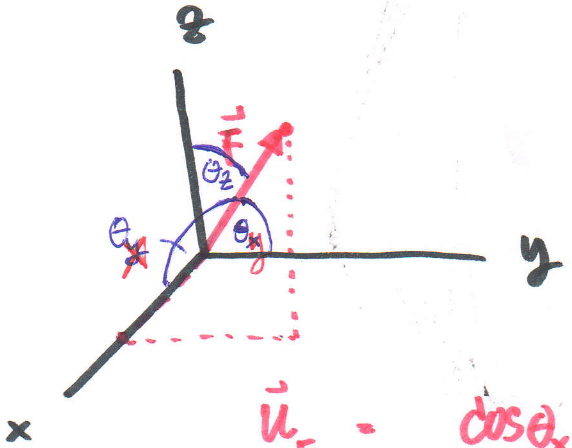
$$\cos\theta_1 = l$$

$$\cos\theta_2 = m$$

สำหรับ การเคลื่อนที่ ในระนาบ 2D

$$l^2 + m^2 = 1$$

<u>3D</u>



$$\vec{u}_F = \cos\theta_x \vec{i} + \cos\theta_y \vec{j} + \cos\theta_z \vec{k}$$

ถ้า

$$\cos\theta_x = l$$

$$\cos\theta_y = m \quad \Rightarrow \quad l^2 + m^2 + n^2 = 1$$

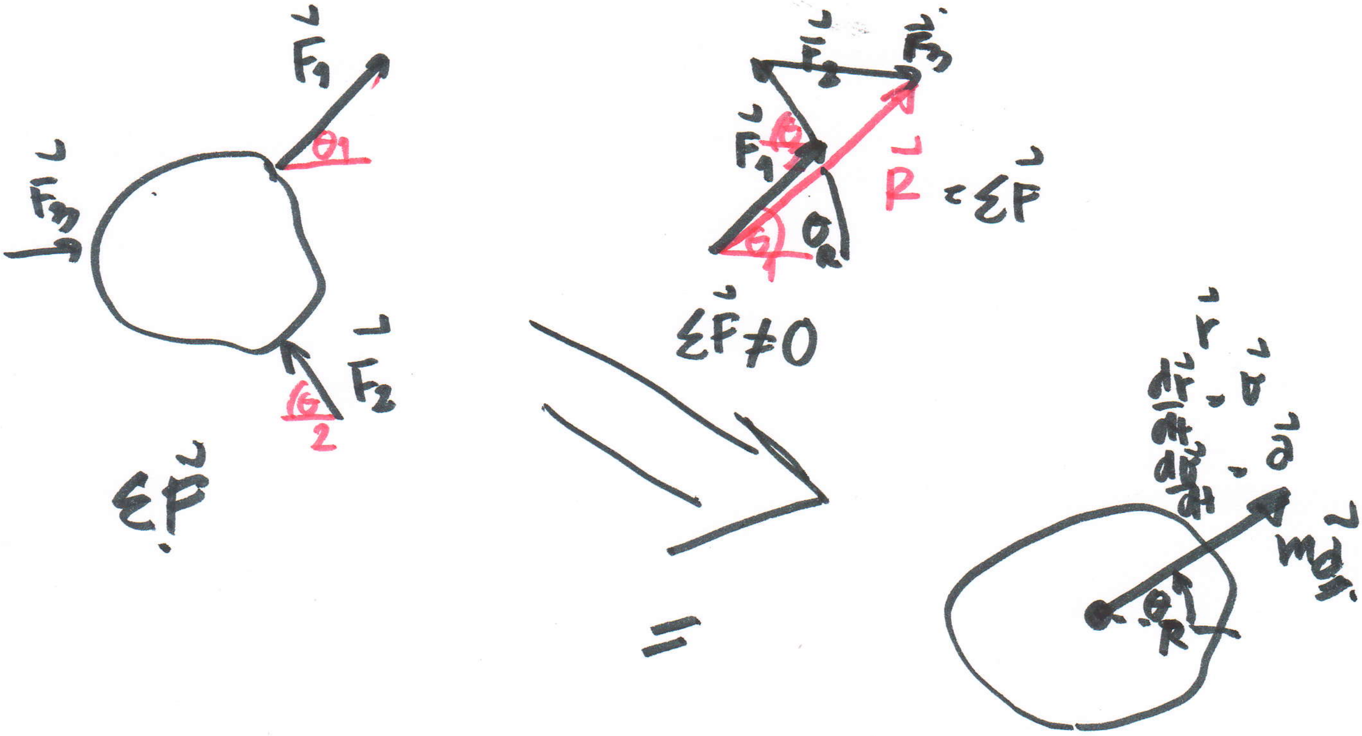
$$\cos\theta_z = n$$

Kinetics

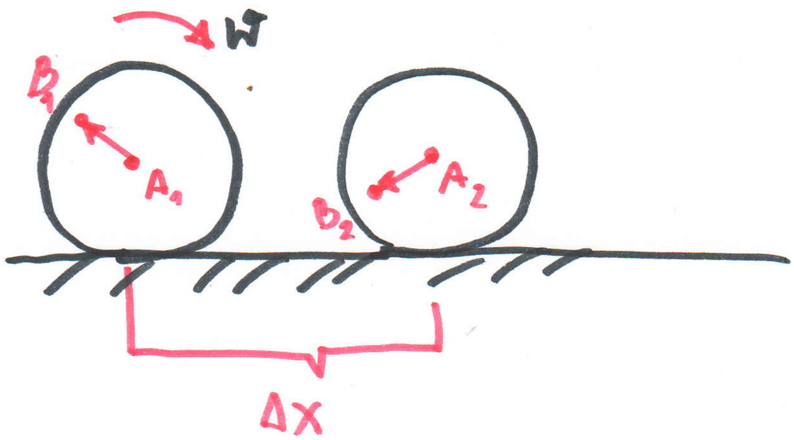
$\sum \vec{F} = m\vec{a}$

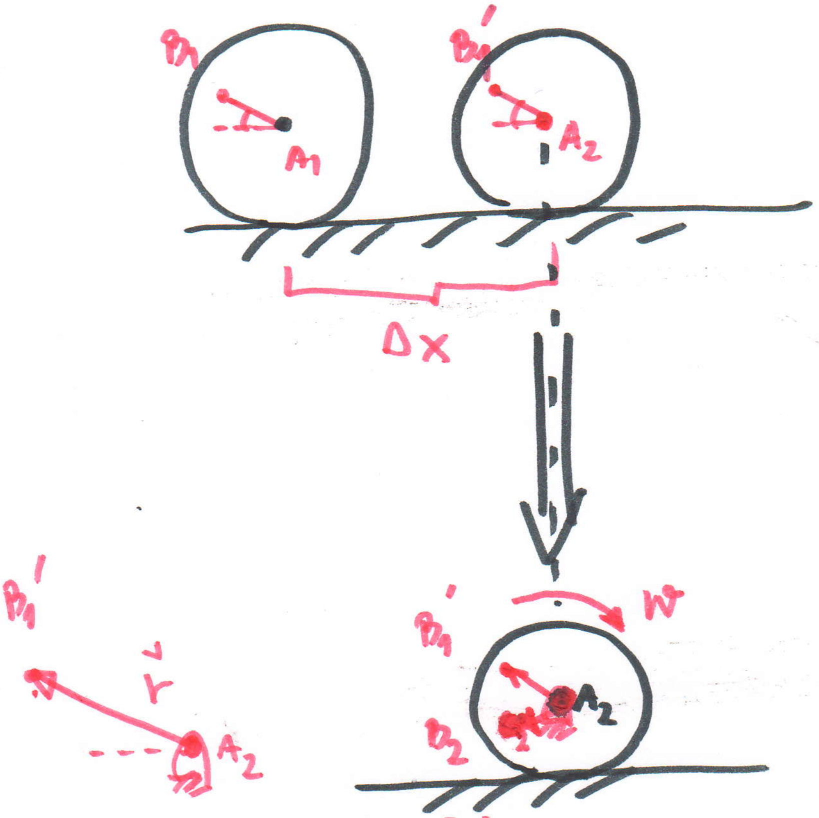
- Kinematics of Rigid Bodies

$\sum \vec{F} = m\vec{a}$ (Kinetics)



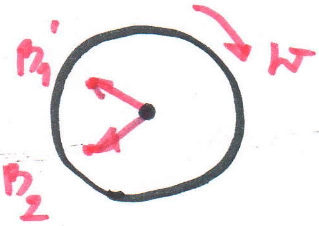
สำหรับการเคลื่อนที่ ของ Rigid Bodies





Translation (การเลื่อน)
ไม่ให้ vector $\vec{AB}$
เปลี่ยนทิศ (ไม่เปลี่ยนมุม)

$$\vec{V}_A = \vec{V}_B$$
$$\vec{a}_A = \vec{a}_B$$



Rotation

$$\vec{v} = \vec{\omega} \times \vec{r}$$
$$\vec{v}_B = \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$\vec{\omega}$ = Angular Velocity
$\vec{\alpha}$ = Angular Acceleration

$$\frac{d\vec{v}}{dt} = \frac{d}{dt}[\vec{\omega} \times \vec{r}]$$
$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$
$$= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$
$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) \text{ ; plane motion}$$
$$\vec{a} = \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}$$

5/7

General Plane Motion = Translation + Rotation about a fixed axis

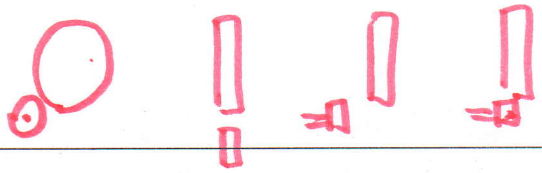
$$\vec{V}_B = [\vec{V}_B]_{\substack{\text{Translation} \\ \text{with } A}} + [\vec{V}]_{\substack{\text{Rotation} \\ \text{about } A}}$$

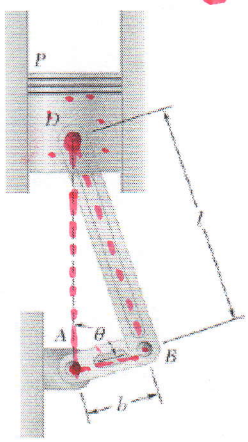
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\boxed{\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\boxed{\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}}$$

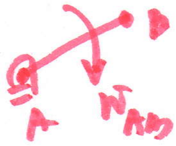




PROBLEM 15.61

In the engine system shown, $l = 160$ mm and $b = 60$ mm. Knowing that the crank AB rotates with a constant angular velocity of 1000 rpm clockwise, determine the velocity of the piston P and the angular velocity of the connecting rod when (a) $\theta = 0$, (b) $\theta = 90°$.

จาก $v_P = v_D$



$\omega_{AB} = 1000$ rpm $= 1000 \times \frac{2\pi}{60} = 104.72$ rad/s

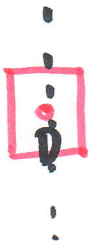
Pistan ⇒ Translation

Crank AB ⇒ Rotation about A

Connecting Rod BD ⇒ General plane motion

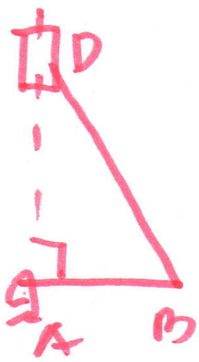
พิจารณาการเคลื่อนที่ของ Rigid bodies แต่ละชิ้น

Pistan



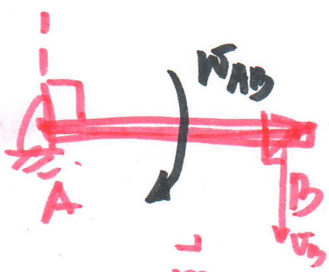
v_D สมมติ ให้ทิศของ ลูกสูบ ↑

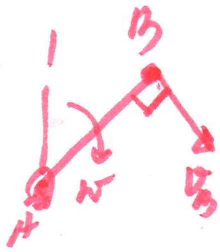
$$\vec{v}_D = v_D \vec{j}$$



Crank AB

$\theta = 90°$





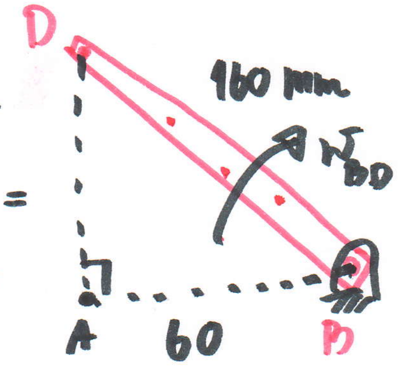
$(\vec{k}\ \vec{i}\ \vec{j})+$

$$\vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$\vec{v}_B = -104.72\vec{k} \times [0.06\vec{i}]$$

$$\vec{v}_B = -6.28\,\vec{j}$$

Connecting Rod BD (General plane Motion)



$\sqrt{160^2 - 60^2} = 148.3$ mm

$$\vec{v}_D = [\vec{v}_D]_{\text{Translation with } B} + [\vec{v}_D]_{\text{Rotation about } B}$$

$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

$$\vec{v}_D = \vec{v}_B + [\vec{\omega}_{BD} \times \vec{r}_{D/B}]$$

$$\vec{v}_D = \vec{v}_B + -\omega_{BD}\vec{k} \times (-0.06\vec{i} + 0.1483\vec{j})$$

$$v_D\vec{j} = -6.28\vec{j} - \omega_{BD}\vec{k}\,[-0.06\vec{i} + 0.1483\vec{j}]$$

$$v_D\vec{j} = -6.28\vec{j} + 0.06\omega_{BD}\vec{j} + 0.1483\omega_{BD}\vec{i}$$

เปรียบเทียบ สัมประสิทธิ์ หน้า i และ j

i : $0 = 0.1483\,\omega_{BD}$

$\omega_{BD} = 0$

j : $v_D = -6.28 + 0.06\,\omega_{BD}$ $=0$

$v_D = -6.28$ m/s

แสดง ทิศทางผิด $\Rightarrow$ $v_D = 6.28$ ↓